

A Finely Crafted O'Brien Unit 2 Opportunity Day

Calculator Section: You may use a calculator. Show all work and circle your answer. Use your time wisely; you will be able to earn additional credit after the timed portion of the test by completing Supercorrections. When you finish, put away your calculator and you can come up to get the non-calculator part- you may continue to work on both sections without your calculator.

1. Determine the amount of time it will take an investment to triple in value if interest is earned at 5%, compounded quarterly.

2. Connor rode his bike 11 miles from Rockport to Thomaston and then completed a 45 mile trip by car from Thomaston to Brunswick. Assume the average rate of the car was 41 mph faster than the average rate of the bike. Find Connor's bike rate if the total time of the trip was 85 minutes.

3. At the right is a "solution" to the equation $100 = 18e^{4k}$.

- a. Check the answer $k \approx 0.398$ back in the equation $100 = 18e^{4k}$ and show that it doesn't work.

$$\begin{aligned}
 100 &= 18e^{4k} \\
 \ln 100 &= \ln(18e^{4k}) \\
 \ln 100 &= \ln 18 \ln(e^{4k}) \\
 \frac{\ln 100}{\ln 18} &= \ln(e^{4k}) \\
 \frac{\ln 100}{\ln 18} &= 4k \\
 \frac{\ln 100}{4 \ln 18} &= k \\
 k &\approx 0.398
 \end{aligned}$$

- b. Circle the error in the "solution," fix that line, and give a correct solution below.

4. Choose values for a and b ($a \neq b$) to make the equation below true.

$$a^{\frac{2}{3}} = b^{\frac{3}{2}}$$

Explain why your equation is true or argue that it is not possible to find such values for a and b .

5. $(x + 2i)$ is a factor of $2x^3 - 3x^2 + 8x - 12$. Find the other two factors.

6. When the polynomial $x^5 + ax + 6$ is divided by $(x - 1)$, the remainder is 9. Find the value of a .

7. Does every polynomial of odd degree have at least one real zero? Why or why not?

8. True or false.

a. $\log_{-2} 4 = 2$

b. $\ln(5 + x) = \ln 5 \ln x$

c. $\log_7 \frac{1}{2} = -\log_7 2$

d. $10^{\log 10^{10}} = 10$

e. $\log x - \log 2 = \frac{\log x}{\log 2}$

f. $\log 4 = \frac{1}{2} \log 16$

g. $\log \left(-\frac{1}{100} \right) = -2$

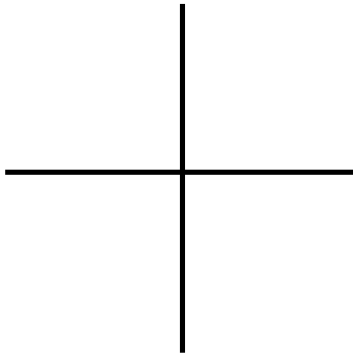
h. $\log_b 8 = \log_b 8 + \log_b 0$

9. The function $g(x) = \frac{x^3 + 2x^2}{x^2 + 4x + 1}$ has a slant asymptote. Find the equation of that asymptote.

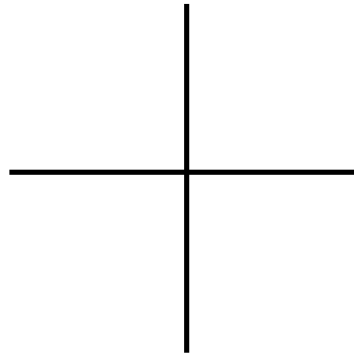
10. Determine the coordinates of a point (a, b) that must be added to the graph of $f(x) = \frac{x^2 - 4}{x + 2}$ to make the function continuous for every real number.

11. Sketch the graph of each function.

a. $f(x) = -\sqrt[3]{x-2}$



b. $g(x) = \frac{2}{x+2} + 2$



12. The diagram shows the graph of the functions y_1 and y_2 . On the same axes, sketch the graph of $\frac{y_1}{y_2}$.

Indicate clearly where the x -intercepts and asymptote occur.



13. Here's your chance to show off your log properties!

a. Use the properties of logs to write the expression as a sum, difference, and/or multiple of logs. Simplify where possible.

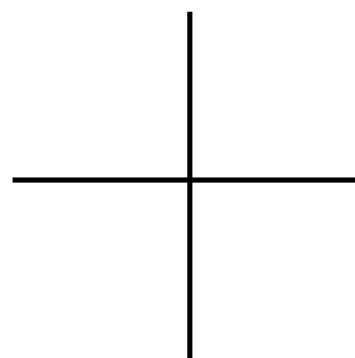
$$\log_4 \frac{16\sqrt{y}}{x^2 z^4}$$

b. Write the expression as the logarithm of a single quantity.

$$\ln 3 + \frac{1}{3} \ln(4 - x^2) - \ln x$$

14. Solve.

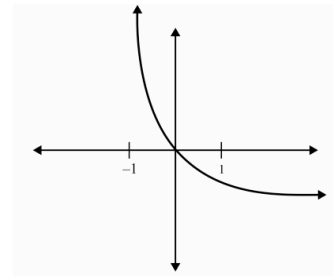
$$\sqrt[3]{x^2 + 3x + 1} = 2$$

15. Given $f(x) = \ln(x + 3)$. Find the x and y intercepts of $f^{-1}(x)$.16. Consider the function $f(x) = \frac{x^3 - 3x^2 - 6x + 8}{x - 2}$. Find its zeros and then sketch its graph.

17. Write a polynomial equation of degree 3 in standard form that has integer coefficients and for which $-3i$ is a root.

18. Consider the graph at right.

a. Give a logarithmic function that could have the graph.



b. Give an exponential function that could have the graph.

19. Choose values for a , b , c , and d so that the graph of $f(x) = ax^3 + bx^2 + cx + d$ crosses the x -axis only at the point $(-1, 0)$.

20. Solve the following equation (Hint: consider using a graphic method).

$$1 + x = 2^x$$

Bonus: Evaluate.

$$9^{\log_3 4} + 4^{\log_3 9}$$